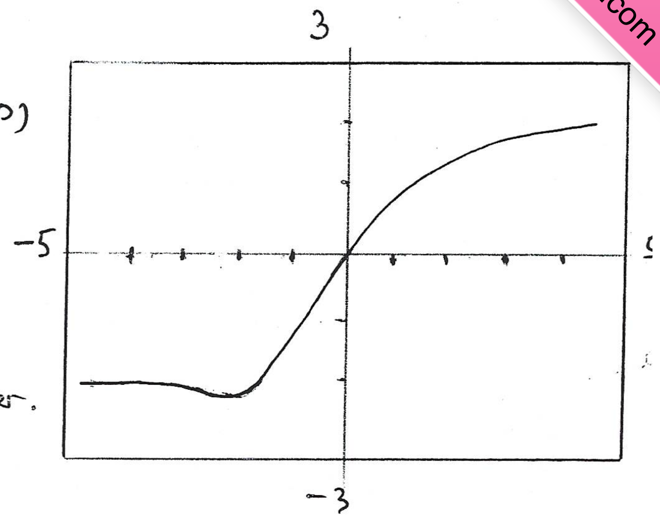
AP Calculus 1995

#1 (a) $y = f(x) = \dfrac{2x}{\sqrt{x^2+x+1}}$

(b)



$x^2+x+1 > 0$

$(x+\frac{1}{2})^2 + \frac{3}{4} > 0$ for all real x

$\Rightarrow$ Domain: $\boxed{\text{All Real } x}$ — Answer.

(c) $\boxed{y = 2, \quad y = -2}$ — Answer.

d) $f'(x) = \dfrac{x+2}{(x^2+x+1)^{3/2}}$

$f'(x) = 0, \quad x = -2$

$f'(x) \quad - \quad \overset{-2}{|} \quad +$ $\Rightarrow$ Rel min^m @ $\left(-2, -\frac{4\sqrt{3}}{3}\right)$

$f(-2) = \dfrac{-4}{\sqrt{3}} = \dfrac{-4\sqrt{3}}{3}$

$\Rightarrow$ $\boxed{\text{RANGE } \left\{ y \mid -\frac{4\sqrt{3}}{3} < y < 2 \right\}}$ — Answer.

NOTE: $\displaystyle\lim_{x\to\infty} \frac{2x}{\sqrt{x^2+x+1}} = \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} = 2$

And $\displaystyle\lim_{x\to-\infty} \frac{2x}{\sqrt{x^2+x+1}} = \lim_{x\to-\infty} \frac{2x}{|x|} = \lim_{x\to-\infty} -2 = -2$

Notice for large x, $\sqrt{x^2+x+1} \to |x|$.

And $\dfrac{2x}{|x|} = -2$ for $x < 0$

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#2 (a) $v(t) = t\cos t$, $t \geq 0$

Particle moves upwards, $v(t) > 0$

when $v(t) = 0$, $t\cos t = 0$

$\boxed{0 \leq t \leq 5}$ $t = 0$, $\cos t = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$

$v(t)$: $+$ on $(0, \frac{\pi}{2})$, $-$ on $(\frac{\pi}{2}, \frac{3\pi}{2})$, $+$ on $(\frac{3\pi}{2}, 5)$; points marked 0, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, 5

Answer: $\boxed{(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 5]}$

(b) $a(t) = \frac{d}{dt} v(t)$

$= 1.\cos t - t\sin t$

$\boxed{a(t) = \cos t - t\sin t}$ Answer

(c) $y(t) = \int v(t)\,dt$

$= \int t\cos t\,dt$

$= t\sin t - \int 1.\sin t\,dt$

$y(t) = t\sin t + \cos t + c$

But $y(0) = 3 \Rightarrow c = 2$

$\boxed{y(t) = t\sin t + \cos t + 2}$ Answer

(d) $t > 0$, $v(t) = 0$

when $t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

when $t = \frac{\pi}{2}$, $y(\frac{\pi}{2}) = \frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} + 2 = \boxed{\frac{\pi}{2} + 2}$ Answer.

#3 $-8x^2 + 5xy + y^3 = -149$

(a) $\frac{d}{dx}\left\{-8x^2 + 5xy + y^3\right\} = \frac{d}{dx}(-149)$

$-16x + 5y + 5xy' + 3y^2y' = 0$

$\boxed{y' = \frac{16x - 5y}{5x + 3y^2}}$

(b) $y - y_1 = m_T(x - x_1)$

$m_T = \frac{16(4) - 5(-1)}{5(4) + 3(-1)^2} = \frac{69}{23} = 3$

$\Rightarrow y + 1 = 3(x - 4)$

$y + 1 = 3x - 12$

$\boxed{3x - y = 13}$

(c) $x = 4.2$, $y = K$

$\Rightarrow 3(4.2) - K = 13 \Rightarrow \boxed{K = -0.4}$ Answer

(d) If $(4.2, K)$ is on the curve

$\Rightarrow -8(4.2)^2 + 5(4.2)K + K^3 = -149$

$-141.12 + 21K + K^3 + 149 = 0$

$K^3 + 21K + 7.88 = 0$

(e) Using graphic calculator

First estimate $K = -0.4$

$\boxed{K = -0.373 \text{ to } 3\text{dp}.}$

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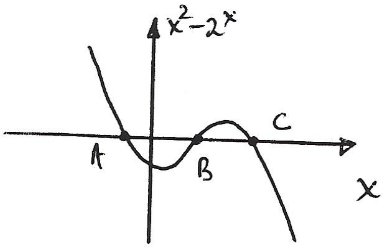
(a) Solve $x^2 = 2^x$

i.e. $x^2 - 2^x = 0$

Using Graphic Calculator:



$x = -0.767$ @ A

$x = 2$ @ B

$x = 4$ @ C

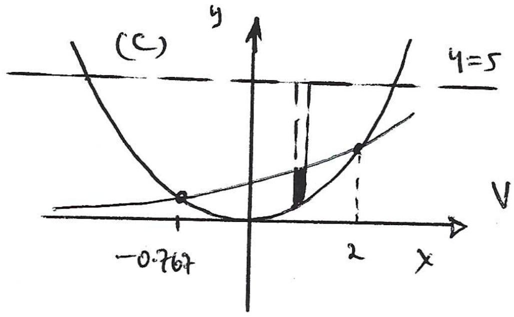
Points of Intersection

$\boxed{(-0.767, 0.588);\ (2,4);\ (4,16)}$

Answer

(b) $A = \int_{-0.767}^{2} 2^x - x^2\,dx + \int_{2}^{4} x^2 - 2^x$

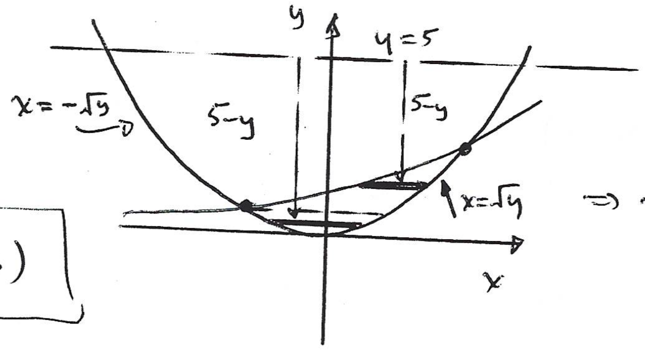
(c) Disc



$V = \pi \int_{-0.767}^{2} (5 - x^2)^2 - (5 - 2^x)^2$

~ OR ~

Shell



$y = 2^x \Rightarrow x = \frac{\ln y}{\ln 2}$

$V = 2\pi \int_{0}^{0.588} (5-y)(2\sqrt{y})\,dy + 2\pi \int_{0.588}^{4} (5-y)\left(\sqrt{y} - \frac{\ln y}{\ln 2}\right)d$

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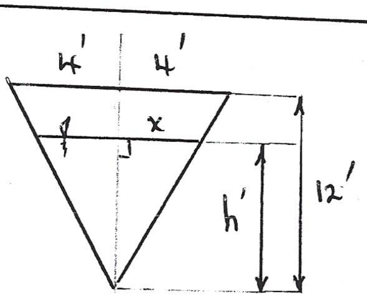


a) Cone

Volume water $= V = \frac{\pi x^2 h}{3}$

$V = \frac{\pi \left(\frac{h}{3}\right)^2 h}{3}$ since $\frac{4}{12} = \frac{x}{h}$

$x = h/3$ (Similar Δ's)

$\boxed{V = \frac{\pi h^3}{27}}$ Answer.

(b) $\frac{dV}{dt} = \frac{3\pi h^2}{27} \cdot \frac{dh}{dt} = \frac{\pi h^2 (12-h)}{9}$

When $h = 3$, $\boxed{\frac{dV}{dt} = -9\pi \text{ ft}^3/\text{min}}$

(NB. Negative sign indicates loss of water)

(c) Large Tank.

$V = 400\pi y$

$\frac{dV}{dt} = 400\pi \frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{9\pi}{400\pi} = \boxed{\frac{9}{400} \text{ ft/min}}$

Ans...

#6 (a) $h(1) = \int_1^1 f(t)\,dt = \boxed{0}$ — Answer

(b) $h'(4) = \frac{d}{dx}\int_1^4 f(t)\,dt = f(4) = \boxed{2}$ — Answer.

(c) $h(x)$ is concave upward when $h'(x)$ is increasing.

In this case when $f(x)$ is increasing.

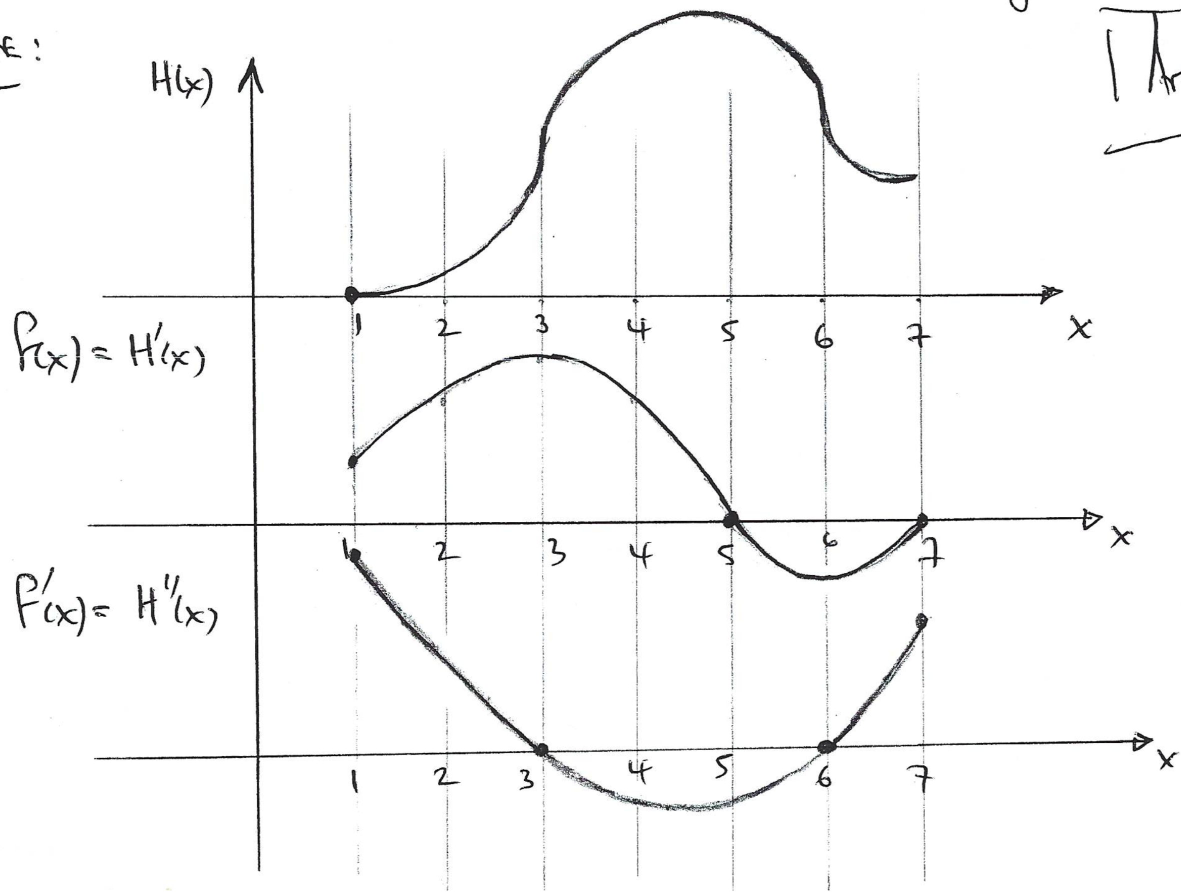
$\boxed{(1,3) \cup (6,7)}$ — Answer.

(d) $h(x)$ has no relative minimum on $[1,7]$ since $h'(x)$ does not change sign from negative to positive.

End points: $h(1) = 0$

$h(7) > h(1)$ since $h(x)$ has a relative maximum at $x=5$ and cannot decrease to zero.

NOTE:



Answer for $x=1$